ANALYSIS OF SHOCK-WAVE REFLECTION FROM A CURRENT LATTICE

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 6, pp. 123–129, 1968

The interaction of a strong shock wave with the magnetic field of a current lattice is analyzed theoretically in the approximation of one-dimensional nonsteady magnetohydrodynamics. The calculated results are compared with experimental data [1].

The basic assumptions of the calculation are: 1) the approximations of one-dimensional nonsteady magnetohydrodynamics are adopted; 2) the gas is assumed ideal with constant coefficients, and dissociation and ionization are neglected; 3) of the dissipative processes in the gas, only Joule heating is taken into account. Certain other assumptions will be stated during the solution of the problem.

The interaction situation can be described in the following manner for the theoretical analysis: we assume that a shock wave is moving from left to right along the x-axis. In the plane x = 0 (Fig. 1), there is an ideally conducting "wall" (a lattice) which is totally permeable to a nonconducting gas. At t = 0, a current of surface density i begins to flow in the lattice in the y-direction.

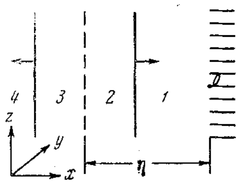

Fig. 1

At this time the shock wave is at a distance $x = -l$ from the lattice. Since the shock-wave velocity is much smaller than the velocity of light c, and since the time lc^{-1} is small in comparison with the characteristic time for the interaction, we assume that a constant magnetic field $H_{00} = 4\pi c^{-1} i$, directed along the z-axis, is established between the shock wave and the lattice at t = 0.

If the gas behind the shock wave is conducting, a current will flow in it in the direction opposite that in the lattice, and a retarding force will arise. As a result of this "collision" between the shock wave and the magnetic field, a reflected shock wave arises, while the weakened incident wave (the refracted wave) penetrates into the magnetic field. The particles at the position of the incident shock wave at t = 0 form a contact surface separating the gas regions compressed by shock waves of different intensity. As the incident shock wave approaches the lattice, the magnetic field intensity and the shock wave velocities change, so that the gas parameters (including the entropy) are functions of x and t in the regions between the incident wave and the contact surface (region 2 in Fig. 1) and between the contact surface and the reflected shock wave (region 3). In front of the incident wave (region 1) and behind the reflected wave (4), the gas parameters are constant.

To determine the nature of the flow, we must solve the nonsteady equations of magnetohydrodynamics in regions 2 and 3 with an account of the relations at the shock waves and at the contact surface. To simplify this problem, we make several assumptions in addition to those stated above: 1) the gas parameters in regions 2 and 3 are functions of only the time; we define them as certain averages over x; 2) the incident shock wave is so strong, and the gas conductivity in region 3 so large, that the magnetic field penetrates only a short distance into region 3, and the reflected shock wave is always ahead of the magnetic field diffusion and is therefore purely gasdynamic; 3) the gas conductivity in region 2 is small; the magnetic field is constant here and is a function of only the time; and the refracted shock wave is gasdynamic.

The first assumption permits a significant simplification of the solution, while retaining much information about the interaction. The second and third assumptions can be justified to a certain extent. The reflected shock wave will be purely gasdynamic in two limiting cases: a) when there is a large conductivity to the left of the contact surface, and when the reflected shock wave overtakes the magnetic field diffusion; b) when the conductivity is small, i. e., when the gas diffusion has a weak effect on the nature of the magnetic field distribution. In a sufficiently high initial magnetic field, the refracted shock wave is attenuated as it moves; the gas behind it may become nonconducting; and the shock wave will also be purely gasdynamic.

Accordingly, the assumption that all the shock waves are gasdynamic will give a more valid description of the strong interaction of a shock wave with the magnetic field of a current lattice. Analysis in this approximation is valid until the contact surface reaches the plane of the current lattice.

Taking all these assumptions into account, we obtain a system of equations describing the interaction. At each shock wave, we have three equations (using the equation of state); we have nine equations in all. At the contact surface, we have the two equations

$$u_3 = u_2, \qquad P_3 = P_2 + PH_0^2 \qquad \left(H_0^2 = \int\limits_{-\infty}^{\eta} jHdx\right). \tag{1}$$

Here H_0^2 is equal to the force per unit area of the contact surface, whose coordinate is denoted by η; and j and H are the current density and magnetic field intensity to the left of the contact surface. The numerical subscripts denote the regions to which the given quantities refer.

Accordingly, we now have 11 equations; let us determine the number of unknowns.

In region 1, ahead of the incident wave, all the parameters are given (including the velocity of this wave). There are two parameters (the pressure p and the density ρ) remaining unknown in each of regions 2, 3, and 4, in addition to the front velocities u_0 and u_n of the reflected and refracted shock waves, the gas velocities in these three regions, and the magnetic field $H_0(t)$. There are thus 12 unknowns.

We can find a twelfth equation by treating the currents in the gas and in the lattice as a common electric loop and by assuming that these currents are shorted at a channel height d along the y-axis by ideal conductors. The equation and initial condition for this electric loop are

$$\frac{d}{dt}\left(H_0\eta + \int\limits_{-\infty}^{\eta} Hdx\right) + \frac{RH_0}{R_m} = E(t), \qquad H_0 = 1 \quad (t = 0). \tag{2}$$

Here R is the resistance per unit area of the gas layer to the left of the contact surface. Equations (1) and (2) are written in dimensionless form; the characteristic quantities are l for the length, u for the velocity, lu^{-1} for the time, $\rho_1 u^2$ for the pressure, ρ_1 for the density, H_{00} for the field, $(c/4\pi)H_{00}l$ for the current, and $(\sigma_4 l)^{-1}$ for the resistance;

$$P = \frac{H_{00}^2}{8\pi\rho_1 u^2}, \qquad R_m = \frac{1}{c^2} 4\pi\sigma_4 ul.$$

The position of the contact surface is given by

$$\eta = -1 + \int\limits_0^t u_3 dt. \tag{3}$$

The quantity E(t) was introduced in Eq. (2) to compensate for the two-dimensional nature of the actual experiment [1]. In the theoretical scheme, the currents passing through the gas and the lattice are equal, and the time dependence of the magnetic field between the gas and the lattice is determined only by their interaction. In the experimental apparatus of [1] there are two electric loops: 1) the loop consisting of the gas, electrodes, and lattice, and 2) that consisting of the capacitors, busbars, and lattice. The lattice is common to both loops; accordingly, the currents in the gas and in the lattice are not equal because the situation is not one-dimensional. The current in the lattice is greater than that in the gas, and it maintains an additional magnetic field which does not depend on the interaction. In the one-dimensional scheme, this can be compensated, at least qualitatively, by introducing some emf E(t) which can maintain a specified current and magnetic field in the loop, independent of the interaction.

In Eq. (2), two new quantities have been introduced: R and H. The resistance of the gas-electrodes-lattice loop is found from the equality of the Joule-heating losses:

$$RI^2 = RH_0^2 = \int_{-\infty}^{\eta} j^2/\sigma_3 \cdot dx$$

(we neglect energy dissipation in the electrodes and in the lattice).

To determine field H, we must solve the equation for magnetic field diffusion in region 3:

$$\frac{\partial H}{\partial t} = -u_3 \frac{\partial H}{\partial x} + \frac{1}{R_m \sigma_3} \frac{\partial^2 H}{\partial x^2} \tag{4}$$

with the initial and boundary conditions

$$H(0, x) = 0, \quad H(t, \eta) = H_0, \quad H(t, -\infty) = 0.$$

We introduce the new independent variables

$$\zeta = \int_0^t \frac{1}{\sigma_3} dt, \qquad \xi = x + 1 - \int_0^t u_3 dt,$$

and we seek a solution of Eq. (4) in the form

$$H = \frac{1}{2\sqrt{\pi R_m}} \int_0^{\zeta} \xi \left(\frac{\zeta - \tau}{R_m}\right)^{-3/2} \exp\left(-\frac{\xi^2 R_m}{4(\zeta - \tau)}\right) H_0 d\tau.$$

This equation for H prevents us from writing simple expressions for R (the gas resistance in region 3) and for the magnetic flux, so we make two more assumptions which are valid for a qualitative analysis of the interaction. First we assume that $\sigma_3 = 1$ and that H_0 changes so slowly as a function of the time that it can be taken through the integral. Then we have

$$H = H_0 \left[1 - \frac{2}{\sqrt{\pi}} \int_0^{\beta} e^{-\alpha^2} d\alpha\right], \qquad \beta = \frac{\xi}{2} \left(\frac{R_m}{t}\right)^{1/2},$$

$$R = \left(\frac{R_m}{2\pi t}\right)^{1/2}, \qquad \int_{-\infty}^{0} H d\xi = 2H_0 \left(\frac{t}{R_m}\right)^{1/2}. \tag{5}$$

Substituting (5) into (2), and making the last assumption—that all the shock waves are strong—we find the system

$$\frac{d}{dt}\left[H_0\left(-\eta + 2\left(\frac{t}{\pi R_m}\right)^{1/2}\right)\right] + H_0 (2\pi R_m t)^{-1/2} = E(t),$$

$$\eta = -1 + \frac{\varkappa + 1}{2\varkappa} \int_0^1 u_n dt, \qquad p_3 = u_n^2 + PH_0^2, \tag{6}$$

$$u_n p_3 = p_3 - (p_3 - 1)\left(\frac{\varkappa - 1}{2\varkappa}(p_3 - 1)\right)^{1/2},$$

with the initial condition $H_0(0) = 1$.

System (6) has been solved on a computer for a ratio of specific heats equal to $\varkappa = 1.67$, for various values of the parameters P and R_m, and for the values

$$E_1 = 0, \qquad E_2 = \frac{1 + \sqrt{2}}{\sqrt{2\pi R_m t}}.$$

We have set $E_1 = 0$ here to correspond to the initial current in our electric circuit and to the fact that $H_{00} = 1$; this field is subsequently governed only by the interaction of the shock wave with the lattice.

In the second case, we have an emf E_2 which depends on the time in such a manner that when $\eta = -1$, it compensates for the magnetic diffusion into the gas and maintains the magnetic field H_{00} constant ($H_{00} = 1$). This specification of the emf is not quantitatively the same as the time dependence of the magnetic field due to the lattice current in the experiments of [1], but it permits us to ascertain the general effect of this field on the interaction.

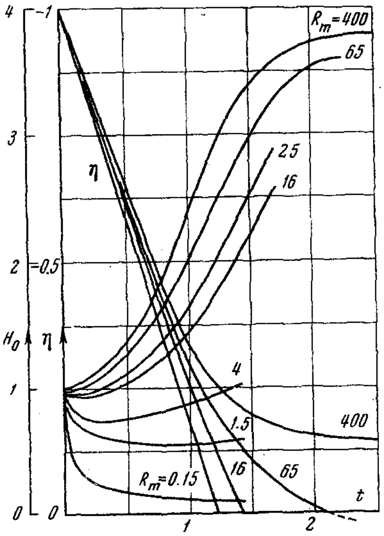

Fig. 2

Figures 2 and 3 show the time dependence of the magnetic field H_0 and the position η of the contact surface for $P = 0.5$ and for two functions E (the parameter is the magnetic Reynolds number R_m). For $E = 0$ (Fig. 2), the initial field diffuses rapidly at small R_m into the gas and, since the interaction amplifies the field slightly under these conditions, H_0 decreases. At some R_m, the magnetic field begins to decrease, and then abruptly increases; however, up to $R_m = 4$, the magnetic field at $\eta(t) = 0$ does not reach its initial value $H_0 = 1$. For $R_m \geq 50$, the magnetic field increases greatly, reaching near the lattice a maximum ($\eta(t) \approx 0$) which is three to four times as great as the initial field $H_0 = 1$. At $R_m \approx 400$, the magnetic field reaches saturation, and remains almost constant near the lattice; the magnetic pressure is essentially equal to the gas pressure behind the reflected shock wave (Figs. 4 and 5). Figures 6 and 7 show the η dependences of the velocities u_n and u_0 of the refracted and reflected shock waves. In Figs. 2, 4, 6, and 8, we have $E = E_1$; in Figs. 3, 5, 7, and 9, we have $E = E_2$). At large R_m, the refracted wave for $\eta = 0$ approaches a sound wave, while the reflected wave becomes so intense that it begins to move relative to the walls, opposite the incident gas flow (positive values of u_0).

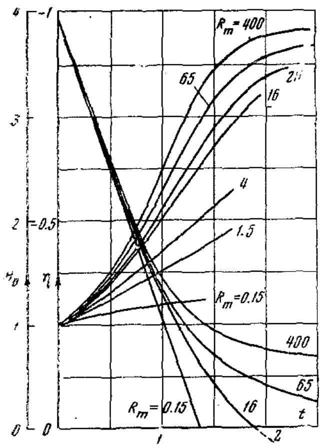

Fig. 3

For $E = E_2$ (Fig. 3), the field decrease observed at $E_1 = 0$ does not occur at small R_m. At large R_m, the difference in the $H_0(t)$ dependences for E_1 and E_2 is noticeable only during the initial stage of the interaction; here the magnetic field is governed only by the interaction itself. With E_2, the interaction is greater for all R_m than in the case $E = 0$ (Figs. 2–7).

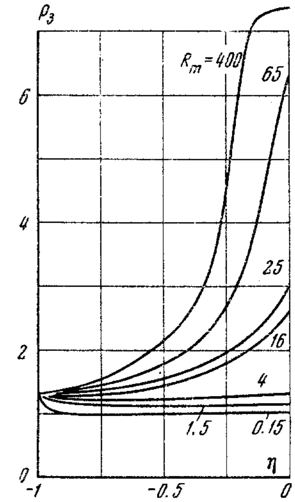

Fig. 4

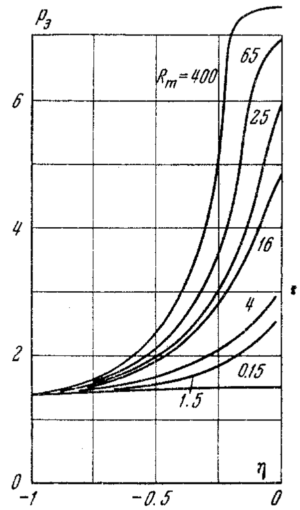

Fig. 5

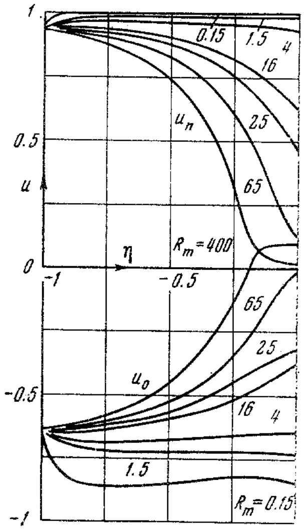

Fig. 6

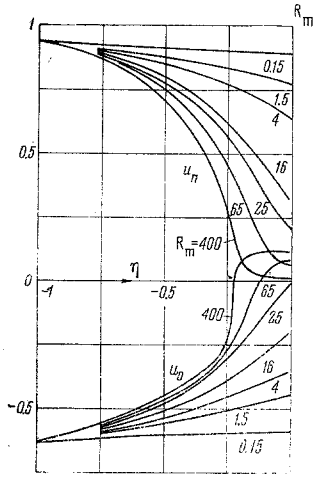

Fig. 7

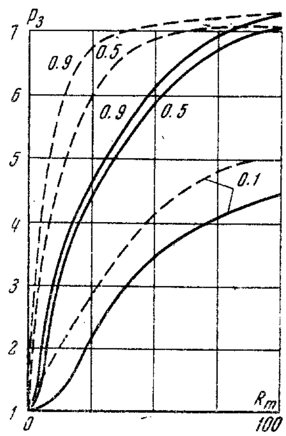

Fig. 8

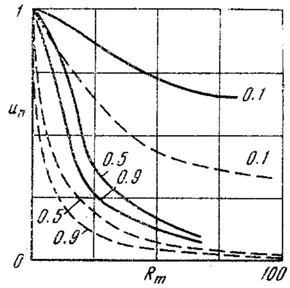

Fig. 9

Figures 8 and 9 show, for $\eta(t) = 0$, the R_m dependences of the pressure p_3 behind the reflected shock wave and the velocity u_n of the refracted wave for various values of the parameter P (the curves are labeled with the P values). As R_m and P increase, there is a sharp intensification of the interaction. There is a certain saturation evident for $P > 0.5$ and $R_m > 50$ (for E_2) or $R_m > 100$ (for E_1). Under these conditions, the pressure p_3 is approximately equal to the pressure behind the reflected shock wave when there is complete retardation of the flow, and the velocity of the refracted wave is approximately equal to the velocity of sound.

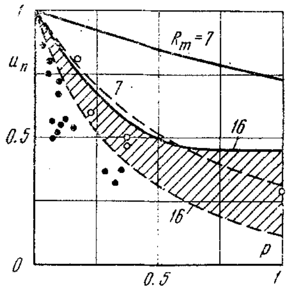

Fig. 10

In Fig. 10, we have attempted to compare the results of this calculation with the experimental data of [1]. This figure shows the dependence of the velocity of the refracted wave on the hydromagnetic parameter P.

The theoretical value of u_n is adopted for a contact-surface position $\eta(t) = 0$, while the experimental value is adopted for the instant at which the refracted wave approaches the end wall. The experimental points [1) $R_m = 12.5$; 2) $R_m = 15$] in Fig. 10 lie below the theoretical values for the given R_m values; the difference is particularly noticeable at $E_1 = 0$. In the experiments, the shock wave approached the electrodes at a time t_1 approximately equal (see Fig. 5 of [1]) to the maximum current from the capacitors through the lattice. During the course of the interaction, this current subsequently decreased, so the magnetic field it produced did also. In the theoretical analysis, the specification $E_1 = 0$ corresponds to the absence of this field, which is produced by the current from the external capacitor bank at $t > 0$; with $E = E_2$, it corresponds to a constant field with $\eta = -1$, i. e., to the presence of a stronger external field than in the experiments. For this reason, the experimental points in Fig. 7 [1] should lie above the curve for $R_m = 16$ and E_2. The assumption $\sigma = \text{const}$ adopted for the calculation apparently has affected the quantitative disagreement between theoretical and experimental data. An account of the gas-temperature dependence of σ, the increase of σ accompanying the Joule heating, and the possibility (mentioned in [1]) of T-layer formation [2] would have caused an increase in the effect of R_m and a significant intensification of this interaction, and would have resulted in better agreement between experimental and theoretical results.

The author thanks S. P. Kurdyumov, V. S. Sokolov, V. A. Derevyanko, and E. F. Lebedev for discussion of this study; and G. I. Sharova and S. P. Mymrina for carrying out the calculations and displaying the results.

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20 March 1968

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